

Additional resources for the teacher

Eratosthenes of Cyrene (about 275-193 BC) is one of the most representative figures of Alexandria in the 3rd century. The interdisciplinary character of Alexandrian science is reflected in his work (mathematics, literary criticism, philosophy, history, geography). In his treatise "On the Dimensions of the Earth" Eratosthenes presents for the first time an innovative method of calculating the earth's circumference.

The story takes place in Egypt, with Ptolemy III the Benefactor as King, around the third century BC. Every year on June 21, in an Egyptian city called Syene, its inhabitants celebrated the arrival of summer. But, beyond the celebration, the inhabitants had discovered a remarkable fact: at noon on the day of the summer solstice (June 21), the Sun is reflected in the water of the well.

Eratosthenes, starting from this impressive event, managed to formulate and realize the amazing idea of measuring the length of the Earth's circumference by applying the "experimental method of research". experiences, assumptions and scientific knowledge of his time:

- It accepts the assumption that the Earth is round (sphere).
- He searches in the Library of Alexandria for the geographical and astronomical knowledge of the time, relevant to the problem that concerns him, together with the methods of obtaining them from the earlier scientists.
- He recalls rudimentary knowledge of geometry as a connoisseur of "Euclid's Elements" and the works of his friend Archimedes.
- He does not fail to inform King Ptolemy and convince him of how important the result of his venture would be for him as well.

Today, an observer located

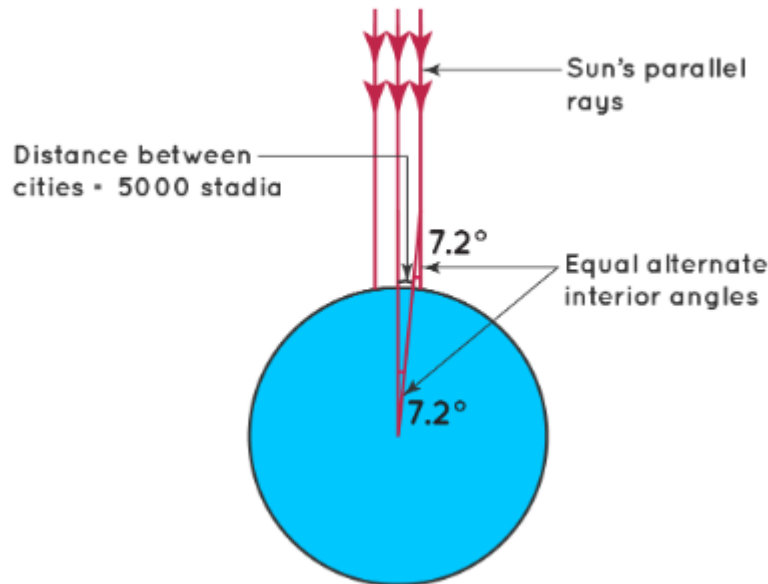
- at the Equator will see the Pole Star on the horizon line,
- at 45 degrees latitude will see the Pole Star 45 degrees above the horizon,
- at the North Pole will see the Pole Star directly above it
- in the Southern Hemisphere the Pole Star is never visible.

Aristotle observed that during a lunar eclipse (when the Sun, Earth, Moon are in the same line) the shadow of the Earth gradually shifts over the surface of the Moon always forming a circular arc.

When a sailboat moves away, its hull (hull) disappears first and its mast last, regardless of the direction in which it is moving. Thus, Eratosthenes ensured that he could rely on his hypothesis of the sphericity of the Earth and proceeded on his own ambitious quest "to measure the length of the circumference of the Earth".

A simple geometric figure will help Eratosthenes to continue his thoughts. With the compass he draws a circle representing the meridian passing through Syene and Alexandria. It's a maximum circle and that's what it aims to measure. On the meridian he marks the positions of the two cities.

Circumference of the Earth



Source : <https://www.cuemath.com/geometry/circumference-of-the-earth/>

The event with the well in Syene ensures that the ray from the Sun is directed towards the center of the Earth and draws it. Also, the fact that a ruler nailed vertically in the garden of the library in Alexandria, at noon on the summer solstice festival, assures him that his extension will pass through the center of the Earth and draws it. Now, an angle ϕ is formed which is centered on the meridian circle and goes to the arc Alexandria – Syene .

Continuing his reasoning, he draws the rays of the Sun parallel (as Thales had done to measure the height of a Pyramid), at which point a new triangle is formed with the base and its shadow on the ground.

He very easily finds that the angle ϕ can be measured, since it is transferred to the new triangle (they are angles within alternately parallel straight lines - rays that pass through the cities and the center of the Earth).

If, in fact, he manages to measure the distance between Alexandria and Syene , then he will be able to arrive at the desired solution to the problem with a method different from the previous ones.

He came up with the idea of using the caravans that made this route, with the compensation of course, since he would pay them from his property! So, he assigned the leaders of the caravans the task of measuring: with the help of some slaves "to count how many turns their cart wheel made, to stretch incredibly long ropes along the road, to count steps, etc." [Leyva , 2008].

The result was encouraging, Eratosthenes ascertained that the distance between the two cities is 5,000 stadia. He eagerly awaited the feast of Cyene (June 21) and at noon he made the last necessary measurement:

At Alexandria the angle made by the sun's rays with the pole driven vertically into the ground was found to be equal to $1/50$ of a full circle (about 7.2 degrees).

Therefore, the length of the circumference of the Earth is found by a multiplication:

50 times 5,000 stages = 250,000 stages

One of the most exciting experiments of antiquity was over. The length of the Earth's meridian passing through Alexandria, Egypt was measured to be 250,000 stadia.

EXPERIMENT INSTRUCTIONS

The appropriate time you should take your measurement for each place is calculated from here .

We calculate the tangent of the angle SAT from the ratio X/Y and thus find the angle which is ϕ degrees. The angle ϕ is equal to the central angle TKI. The latitude of our location is ϕ degrees. Note: The angle ϕ is equal to the latitude only if the measurement is made on the days of the vernal or autumnal equinox.

The distance from the TI=S equator is calculated by Google Earth or from here .

The circumference of the Earth and its radius R are calculated using the following mathematical relationships:

Περμ. Γης: Circumference of the earth

$$\text{Περμ. Γης} = \frac{360}{\phi} * S$$

$$R = \frac{\text{Περμ. Γης}}{2 * \pi}$$

1) To find the coordinates of our school (or the place where we will carry out the experiment): Click on the link: <https://www.google.com/maps> and search for the place we want

on the map and double-click on it, copy the coordinates from the address bar (the first number is the latitude and the second is the longitude)

(2) To measure the distance of our school from the equator:

We click on the link: <https://www.nhc.noaa.gov/gcalc.shtml> and give the coordinates of our school and the equator (for the equator the latitude is 0 and the longitude the same as our school), choose unit of measure and we do the calculation.

(3) To find the time when the sun sets: We click on the link:

<http://suncalc.net> we locate the point we want on the map and move the pointer there, select a date and we have the results on the right side of the page.

To calculate the shadow angle:

We click on the link: <https://el.calcprofi.com/trigono-arithmomichani.html> and give the data from our measurements.

(length of rod, length of shadow, contained angle=90°) and we calculate the remaining elements of the triangle.

How do I calculate the Equator?

We pay attention to:

- The rod we will use must be straight.
- The shadow should fall horizontally.
- The length of the rod must be at least one meter.
- The support of the rod must be stable and absolutely vertical.

- Vertical support is checked with a spirit level or spirit level.
- All of the above must contribute to the rod leaving as distinct a shadow as possible.

Convertercorners

Measure Earth's circumference **distance between the schools angular distance between the schools = Earth's circumference 3600**